Opinion dynamics and wisdom under out-group discrimination

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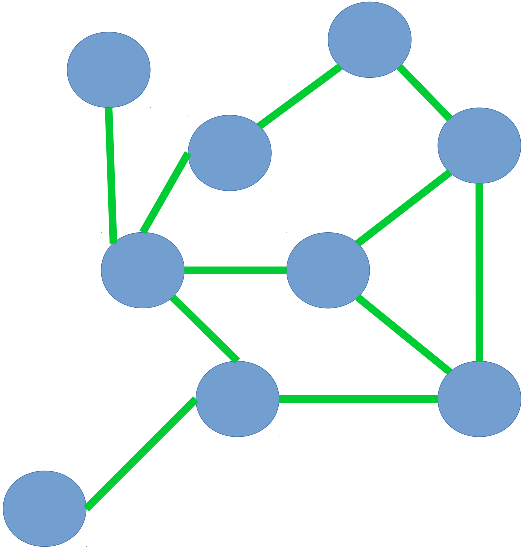
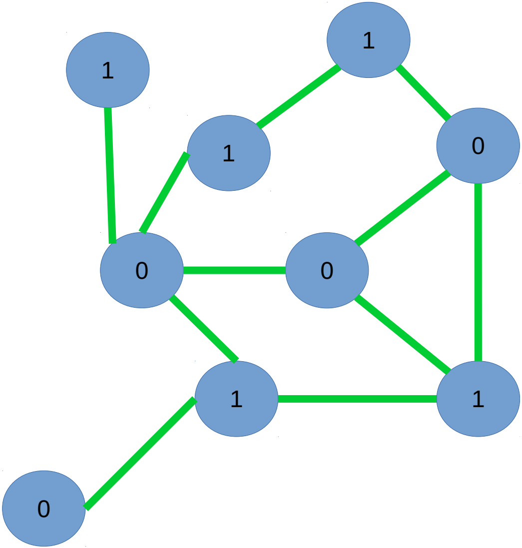
Opinion dynamics

- Models how agents form beliefs and opinions about social/economic/everyday/... issues
- A lot of research from various disciplines such as economics, complexity science/physics, etc.
  - DeGroot 1974, Deffuant et al. 2000, Hegselmann-Krause 2002-2010
  - Golub and Jackson 2012/2014, Acemoglu et al. 2010/2012

Example



Example



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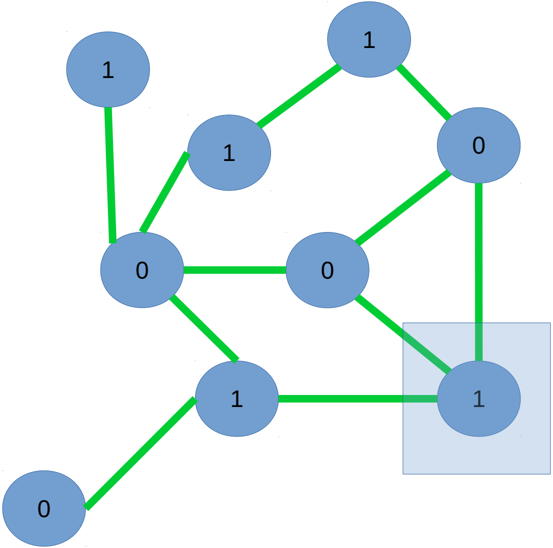

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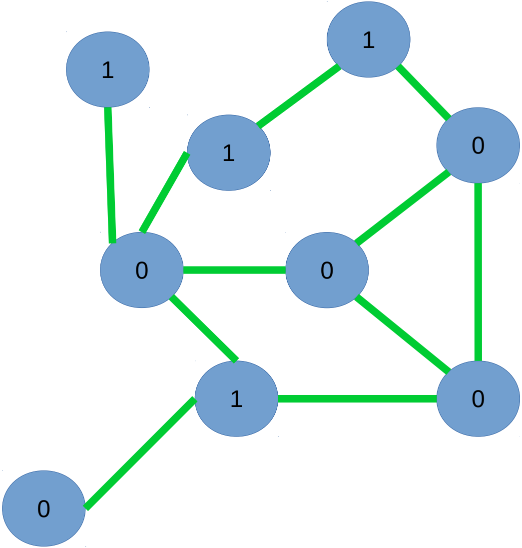
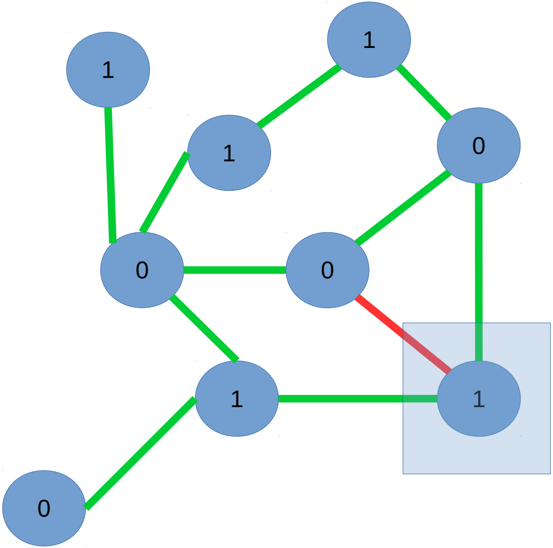


Negative relationships

- More recent works allow *negative* relationships between agents
- Also named *anti-conformity*, *opposition*, *out-group discrimination*, ...
- Relevant works are:
  - Fent, Groeber, Schweitzer 2007: Coexistence of Social Norms based on In- and Out-group interactions
  - Cao et al. 2013: Rebels lead to the doctrine of the mean
  - Altafini 2013: Consensus Problems on Networks With Antagonistic Interactions
  - Javarone 2014: Social Influences in Opinion Dynamics: the Role of Conformity
  - Jarman et al. 2015: The Critical Few: Anticonformists at the Crossroads of Minority Opinion Survival and Collapse
  - etc.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Contributions

- Provide a generalized model of ‘opposition’
- Provide utility function based motivations
- Provide a few mathematical results

Model

- A set of n agents, denoted by $[n] = \{1, \ldots, n\}$
- A set S, the opinion spectrum
  - S is either continuous, e.g., $S = [0, 1]$ (CONTINUOUS MODEL), or
  - S is discrete, e.g., $S = \{\text{Yes}, \text{No}\}$ (DISCRETE MODEL)
- Each agent i has an *in-group* (his 'friends') and an *out-group* (his 'enemies')
- Each agent i also has two associated functions
  - $\mathfrak{F}$ is the identity function on S — for his in-group
  - $\mathfrak{D}_i$ is not the identity function on S — for his out-group. Call $\mathfrak{D}_i$ also *deviation* or *inversion function* (could also make stronger assumptions on $\mathfrak{D}_i$)

Model

- Each agent $i \in [n]$ has a *utility function* for the opinions $b_1, \ldots, b_n$ of all other agents

$$u_i(b_1, \ldots, b_n) = -\sum_{j \in \mathrm{In}(i)} W_{ij}\left(b_i - \mathfrak{F}(b_j)\right)^2 - \sum_{j \in \mathrm{Out}(i)} W_{ij}\left(b_i - \mathfrak{D}_i(b_j)\right)^2$$

- Similar utility function for the `DISCRETE MODEL`, but omitted here

Model

- Agents are utility-maximizers, they choose their opinions b_i to maximize their utility functions given the opinions of their peers
- In the CONTINUOUS MODEL, this leads to the dynamics:

$$b_i(t+1) = \sum_{j \in \mathrm{In}(i)} W_{ij} \mathfrak{F}(b_j(t)) + \sum_{j \in \mathrm{Out}(i)} W_{ij} \mathfrak{D}_i(b_j(t))$$

- In the DISCRETE MODEL, this leads to the dynamics:

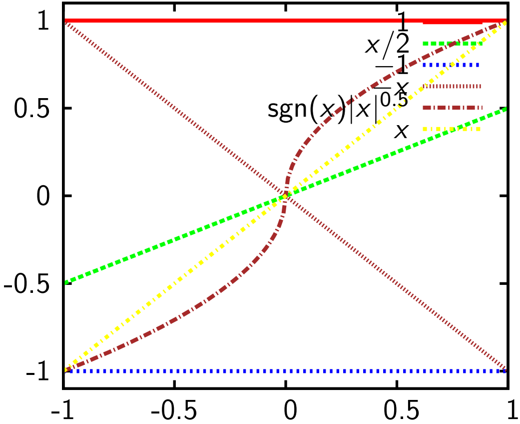
$$b_i(t+1) = \text{weighted majority (inverted) opinion of peers}$$

Example 1

- Let $S = \{0, 1\}$ be a binary opinion space
- Let for all $i \in [n]$: $\mathfrak{D}_i(x) = \mathfrak{D}(x) = \begin{cases} 1 & \text{if } x = 0, \\ 0 & \text{if } x = 1 \end{cases}$
- If $\text{In}(i) = [n]$: agent i always chooses (weighted) majority opinion of his peers $\longrightarrow$ "conformist"
- If $\text{In}(i) = \emptyset$: agent i always chooses (weighted) minority opinion of his peers $\longrightarrow$ "anti-conformist"

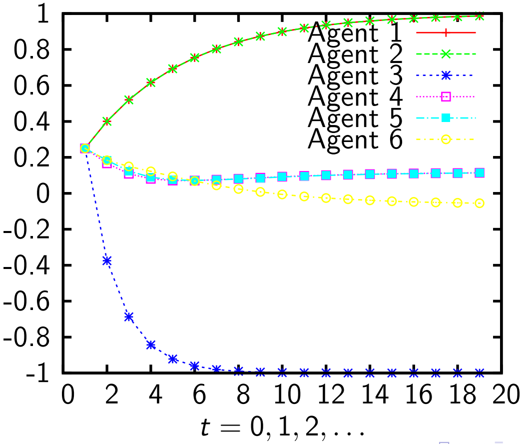
Example 2

- Let $S = [-1, 1]$
- Deviation functions:



Example 2

- Let $S = [-1, 1]$
- Opinion dynamics:



Results 1 — Long-run disagreement

- Under (a few, weak, technical conditions) agents disagree in the long-run whenever

$$\text{Fix}(\mathfrak{D}_i) \cap \text{Fix}(\mathfrak{D}_j) = \emptyset$$

  for two agents i, j
- Here $\text{Fix}(\mathfrak{D}) = \{x \mid \mathfrak{D}(x) = x\}$
- The result means that to agree in the long-run there must exist an opinion x which is “neutral” for all agents $i \in [n]$ in the sense that $\mathfrak{D}_i(x) = x$
- $\longrightarrow$ **long-run/persistent disagreement** is “likely” under our model of out-group discrimination

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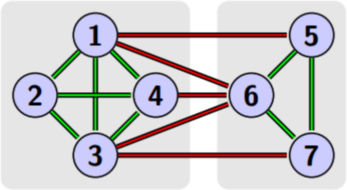
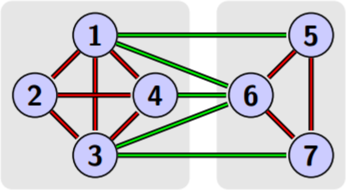
Results 2 — Definitions

Opposition bipartite network



Results 2 — Definitions

Reverse opposition bipartite network



Results 2 — Opinion polarization

Consider the following special case

- $S = \mathbb{R}$
- $\mathfrak{D}_i(x) = \mathfrak{D}(x) = -x$
- Also assume some technical conditions: symmetry of out-group relationship, aperiodicity of graphs wherein agents interact, connectedness of graphs, etc.
- Then, there are only three possible outcomes in the long-run: **consensus**, **polarization** and **non-convergence** (oscillation of beliefs)

Results 2 — Opinion polarization

- Agents' opinions polarize in the long-run iff network wherein they interact is **opposition bipartite**
- Agents' opinions do not converge in the long-run iff **reverse opposition bipartite**
- Agents' opinions reach a consensus in the long-run iff neither **opposition bipartite** nor **reverse opposition bipartite**

Conclusion

- Presented a generalized model of opinion dynamics under out-group discrimination (negative relationships)
- Based on the idea of “deviation functions” and in-group/out-group structure
- Provided utility function based motivation
- Also shown two mathematical results:
    - long-run agreement in society is difficult under our model
    - we also gave network conditions of long-run polarization, consensus and non-convergence in a special case

- Relevant papers:
  - Eger, S. Opinion dynamics and wisdom under out-group discrimination. Mathematical Social Sciences, 2016
  - Eger, S. On limits of powers of certain absolutely row-stochastic matrices. Linear Algebra and its applications, 2016

Thank You!